UDK 539.3.

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FREE DAMPED OSCILLATIONS OF A THREE-LAYER PLATE

Introduction

Three-layer design of various configurations are widely used in aircraft construction. In connection with this urgent task is to study the dynamic characteristics of these systems. The studies which were conducted in the framework of classical theory of thin shells,are mainly aimed at studying the influence of surface conditions on the consolidation of the frequencies and forms oscillations. The general solution of the problem of natural vibrations of isotropic cylindrical shell with consideration of any boundary conditions was proposed in [1].

The qualitative analysis of shapes and natural frequencies of thinelastic isotropic shells and the investigation of the frequency's density problem and the edge effects were studied in [2, 3].

Problems which are arising from the using of the finite element method in membrane's calculationgs are associated with the approximation of the surface and with the volume of calculations. It should be emphasized that, despite the development of numerical methods of calculation, it is necessary to receive the analytical solution, which is accurate.

Statement and solution of the Problem

We consider that a three-layer plate is consisting of two identical outer layers of thickness h with elastic modulus E and the middle layer of thickness 2h with a modulus of elasticity E (Fig. 1.).

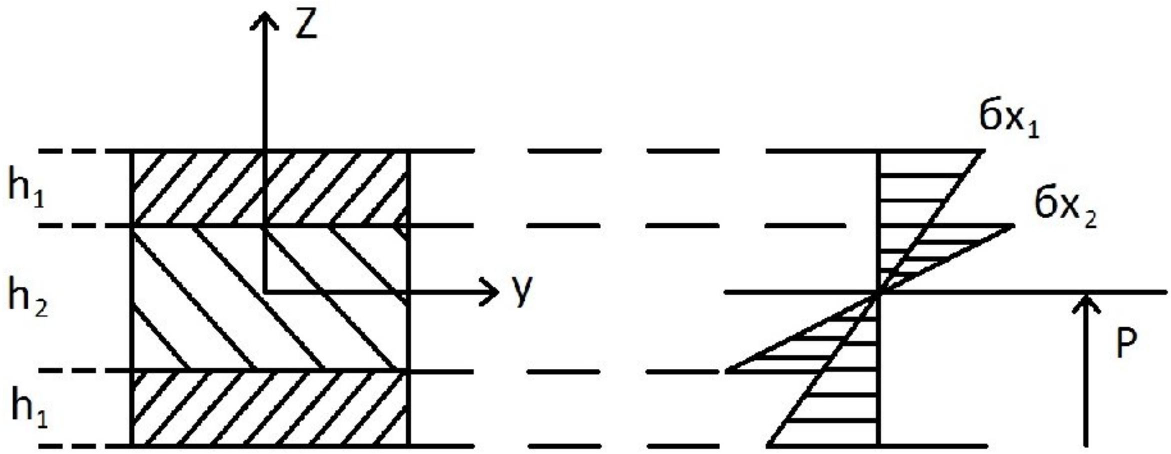

Fig.1. The distribution of stresses in the layers of sandwich plates

We define the reduced stiffness of sandwich plates from the condition of equality of the bending moments for composite plate and solid plate with the same sizes, which was made of a material of the outer layer.

For a solid plate the expression for the bending moment and stress can be written as

$$M_x = -D(W_{xx} + \upsilon W_{yy}),$$

$$\sigma_x = -\frac{E_z}{1-\upsilon^2}(W_{xx} + \upsilon W_{yy}),$$

where υ is the Poisson ratio.

Hence, excepting for the deflection W, we obtain

$$M_x = D\frac{1-\upsilon^2}{E}\cdot\frac{\sigma_x}{z} = D\frac{1-\upsilon^2}{E}\cdot\frac{\sigma_{x1}}{h_1+h_2}.$$

Let's find the relation between the stresses in the layers of the terms of the curvature ρ of the neutral layer of the composite plate

$$\sigma_x = \frac{z}{\rho}E.$$

We have

$$\sigma_{x1} = \frac{(h_1+h_2)}{\rho}E_1, \qquad \sigma_{x2} = \frac{h_2 E_2}{\rho}.$$

Excluding the curvature, we obtain

$$\sigma_{x2} = \frac{h_2}{h_1+h_2}\cdot\frac{E_2}{E_1}\sigma_{x1}.$$

We define the bending moment of tension in the cross section

$$M_x = \int_{-h_2}^{h_2} \frac{\sigma_{x1}}{h_2} z^2 dz + 2 \int_{h_2}^{h_1+h_2} \frac{\sigma_{x1}}{h_1+h_2} z^2 dz = \frac{\sigma_{x2} I_2}{h_2} + \frac{\sigma_{x1} I_1}{h_1+h_2}.$$

Where I_1, I_2 – are the moments of inertia of the outer and middle layers, respectively.

After the substitution (1) and (2) we get

$$D_n = \frac{E_1 I_1 + E_2 I_2}{1-\upsilon^2}.$$

We find the expression of Poisson's ratio for the three-layer structure of a single element:

$$\upsilon_n = \frac{\varepsilon_y^n}{\varepsilon_x}.$$

We are considering the equilibrium of each layer separately as shown in Fig. 2.

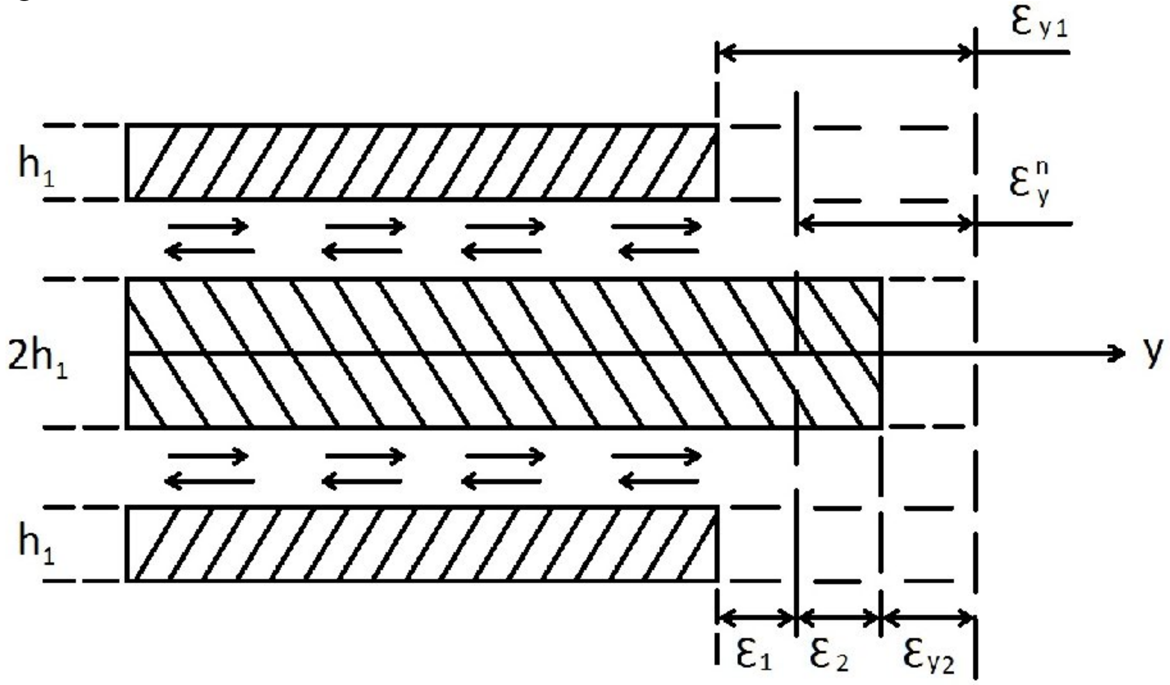

Fig. 2. Displacement in the layers

The geometric relations, as shown in Fig. 2, are

$$\varepsilon_1 = \varepsilon_{y1} - \varepsilon_y^n, \varepsilon_2 = \varepsilon_y^n - \varepsilon_{y2}.$$

Considering Hooke's law for tension-compression

$$\varepsilon = \frac{N}{EF},$$

we obtain relations for the deformation of the layer

$$\varepsilon_2 = \frac{E_1 h_1}{E_2 h_2} \varepsilon_1 .$$

After substituting to the equations the appropriate values and their farther solutions we have

$$\frac{h_2 E_2}{h_1 E_1} = \frac{\upsilon_1 - \upsilon_2}{\upsilon_n - \upsilon_2},$$

finally

$$\nu_n = \frac{\nu_1 E_1 h_1 + \nu_2 E_2 h_2}{E_1 h_1 + E_2 h_2}.$$

The cylindrical rigidity of the sandwich plate and the reduced mass are

$$D_n = \frac{E_1 I_1 + E_2 I_2}{1 - \nu_n^2}, \qquad m_n = \frac{2}{g}(h_1 \gamma_1 + h_2 \gamma_2).$$

Differential equation of the damped oscillations has the form

$$D_n \nabla^2 w + m_n w_{tt} + k w_t = 0$$

coefficient of resistance can be represented as $k = m_n \omega_\gamma$ after substituting in the equation of motion the function of deflections, we have

$$8D_n\left(\frac{3c_1}{a^4} + \frac{c_1 + c_2}{a^2 b^2} + \frac{3c_2}{b^4}\right) - \left(\frac{k^2}{4m_n^2} + \omega^2\right)\left(\frac{x^2}{a^2}c_1 + \frac{y^2}{b^2}c_2 - c_3\right)\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right) = 0.$$

Let's look at the static boundary conditions $x = \pm a;\ y = 0$

$$M_x = -D_n(w_{xx} + \nu_n w_{yy}) = \frac{1}{a^2}(5c_1 - c_3) + \frac{\nu_n}{b^2}(c_1 - c_3) = 0.$$

We obtain $c_1 = \dfrac{b^2 + \nu_n a^2}{5b^2 + \nu_n a^2} c_3 \equiv k_1 c_3$, if $y = \pm b;\ x = 0$

$$M_y = \frac{\nu_n}{a^2}(c_2 - c_3) + \frac{1}{b^2}(5c_2 - c_3) = 0.$$

We obtain

$$c_2 = \frac{a^2 + \nu_n b^2}{5a^2 + \nu_n b^2} c_3 \equiv k_2 c_3 .$$

In the center of of the plate $x = 0; y = 0$ and natural frequency can be determined

$$\omega = \sqrt{\frac{32 D_n}{m_n(\gamma^2 + 4)}\left(\frac{3k_1}{a^4} + \frac{k_1 + k_2}{a^2 b^2} + \frac{3k_2}{b^4}\right)} .$$

Conclusions

We have defined the reduced Poisson's ratio for the three-layer symmetric with respect to the median plane of the plate, we got a three-layer expression of the cylindrical rigidity of the plate, which is composed of the materials with mechanical properties of comparable magnitude. The frequency of free damped oscillations was found.

References

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